Time Dependent Angular Analysis of $B_{s(d)} \rightarrow J/\psi \Phi(K^{*0})$, and a Lifetime Difference in the B_s System

Ke Li (Yale University)

CDF II Collaboration

Aug 28, 2004

DPF 2004, UC Riverside



Outline



- Motivation
- Analysis introduction
- Experimental technique and Fitting Model
- Results and crosschecks
- Conclusion and plans

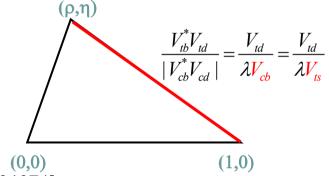
Motivation:



- B mixing
 - \Rightarrow Mass eigenstates $B_{s,H}$ and $B_{s,L}$
 - ⇒ Nearly CP eigenstates
- SM predicts Lifetime Difference in B_s system
 - \triangleright On-shell transitions contribute to $\Delta\Gamma_s$
 - $\frac{\Delta\Gamma_{s}}{\Delta m_{s}} \Rightarrow \Delta m_{d} / \Delta m_{s} \Rightarrow |V_{td}|/|V_{ts}|$ $\frac{\Delta\Gamma_{s}}{\Delta m_{s}} = (3.7^{+0.8}_{-1.5}) \times 10^{-3}, \frac{\Delta\Gamma_{s}}{\Gamma} = 0.12 \pm 0.06$

$$B_s^H = \frac{1}{\sqrt{2}} (|B_s\rangle + |\overline{B}_s\rangle) \rightarrow CP - odd$$

$$B_s^L = \frac{1}{\sqrt{2}} (|B_s\rangle - |\overline{B}_s\rangle) \rightarrow CP - even$$

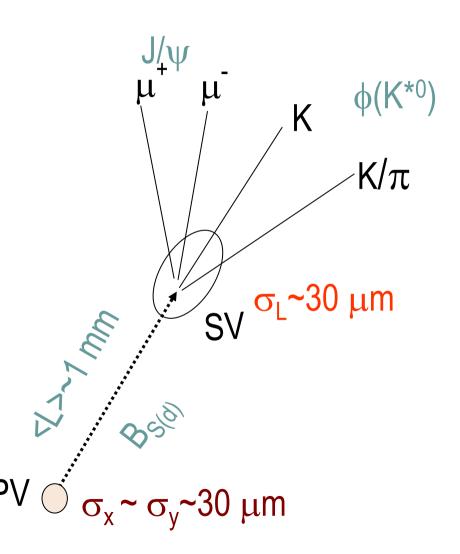


[B Physics at the Tevatron: Run II and Beyond Hep-ph/0201071]

- > $B_{s(d)} \rightarrow J/\psi \phi(K^{*0})$: Pseudoscalar \rightarrow Vector Vector
 - \Rightarrow 0 \Rightarrow 1 \oplus 1, Orbital L = 0, 1, 2 (S, P, D)
 - > Three amplitudes (partial wave, helicity, or transversity basis)
- > Transversity basis: seperates CP (P) odd state nicely.
 - \rightarrow A₀ = S + D wave \Rightarrow CP(P) even
 - \rightarrow A_{II} = S + D wave \Rightarrow CP(P) even
 - \rightarrow A₁ = P wave \Rightarrow CP(P) odd
- \rightarrow Time dependent transversity analysis can isolate the two B states and determine $\Delta\Gamma_s$
- > B_d decays: Sister Channel. Control sample, check if results are sensible.

Event Reconstruction





CDF Run II up to 2/2004

- > L ~ 260 pb⁻¹
- > $J/\Psi \rightarrow \mu^+\mu^-$
 - Muon detector
 - Di-Muon Trigger Path
- \rightarrow $\phi \rightarrow K^+K^-; K^* \rightarrow K^+\pi^-$
 - Well measured in Tracking Chamber (COT)
 - With Silicon Detector Hits
 - Mass window, p_T cut
- > $B_d \rightarrow J/\Psi K^*; B_s \rightarrow J/\Psi \phi$
 - Vertex-fit, p_T cut
- Primary Vertex from Beamline

Transversity Basis



$$egin{aligned} rac{d^4 \mathcal{P}}{dec{
ho}\,dt} &\propto |A_0|^2 \cdot g_1(t) \cdot f_1(ec{
ho}) + \ |A_{\parallel}|^2 \cdot g_2(t) \cdot f_2(ec{
ho}) + \ &\langle A_{\perp}|^2 \cdot g_3(t) \cdot f_3(ec{
ho}) &\Rightarrow \ &\langle A_{\perp}|^2 \cdot g_3(t) \cdot f_3(ec{
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$$egin{array}{lll} f_1(ec{
ho}) &=& 2\cos^2\psi(1-\sin^2\theta\cos^2\phi) \ f_2(ec{
ho}) &=& \sin^2\psi(1-\sin^2\theta\sin^2\phi) \ f_3(ec{
ho}) &=& \sin^2\psi\sin^2\theta \ f_4(ec{
ho}) &=& -\sin^2\psi\sin2\theta\sin\phi \ f_5(ec{
ho}) &=& rac{1}{\sqrt{2}}\sin2\psi\sin^2\theta\sin2\phi \ f_6(ec{
ho}) &=& rac{1}{\sqrt{2}}\sin2\psi\sin2\theta\cos\phi \end{array}$$

$$g_{m{i}}(t)$$
 different for B_d and B_s and are rather non-trivial

A. Dighe et. al., Eur. Phys. J. C 6, 647-662

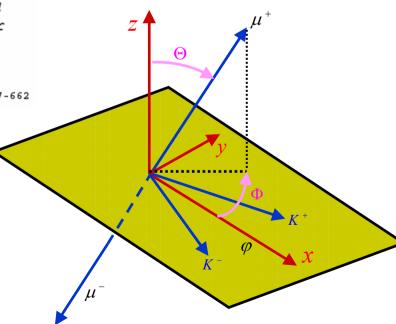
Interference Terms

Transversity angles are defined in J/ψ rest frame

 ϕ (K*0) flight direction \equiv positive **x** KK (K π) plane \equiv **xy** plane

CP(P) Even

CP(P) ODD



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Time-Dependent Distribution



B_s :

$$\begin{split} \frac{d^4\mathcal{P}}{d\vec{\rho}\,dt} &\propto |A_0|^2 \cdot e^{-\Gamma_L t} \cdot f_1(\vec{\rho}) + \\ &|A_{||}|^2 \cdot e^{-\Gamma_L t} \cdot f_2(\vec{\rho}) + \\ &|A_{\perp}|^2 \cdot e^{-\Gamma_H t} \cdot f_3(\vec{\rho}) + \\ ℜ(A_0^*A_{||}) \cdot e^{-\Gamma_L t} \cdot f_5(\vec{\rho}) \end{split}$$

$$\begin{split} &\Gamma_L = CP - \text{even} \\ &\Gamma_H = CP - \text{odd} \end{split}$$

- flavor blind decay
- $\delta\phi_{CPV}\approx 0.03$
- ullet Δm_s is large

B_d :

$$egin{aligned} rac{d^4 \mathcal{P}}{d ec{
ho} \, dt} & \propto \left\{ |A_0|^2 \! \cdot \! f_1(ec{
ho}) + \ & |A_{||}|^2 \! \cdot \! f_2(ec{
ho}) + \ & |A_{\perp}|^2 \! \cdot \! f_3(ec{
ho}) \pm \ & Im(A_{||}^* A_{\perp}) \! \cdot \! f_4(ec{
ho}) + \ & Re(A_0^* A_{||}) \! \cdot \! f_5(ec{
ho}) \pm \ & Im(A_0^* A_{\perp}) \! \cdot \! f_6(ec{
ho})
ight\} \! \cdot \! e^{-\Gamma_d t} \end{aligned}$$

- flavor specific decay
- $\delta \phi_{CPV} = 2\beta$

Fitting Model

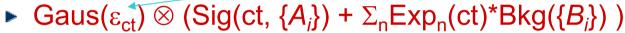


Unbinned likelihood fit, simultaneously fit angular, lifetime and mass distributions

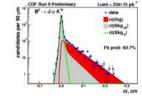
> Mass: Gaus(m, ε_m) + Pol₁(m)

Lifetime + Amplitudes:

Errors on mass and ct are scaled by scaling factor $S_{m,} S_{ct}$ (Floating in the fit)



- Long-lived
 - Displaced J/ψ paired with (random) track
 - Reflections and partially reconstructed B



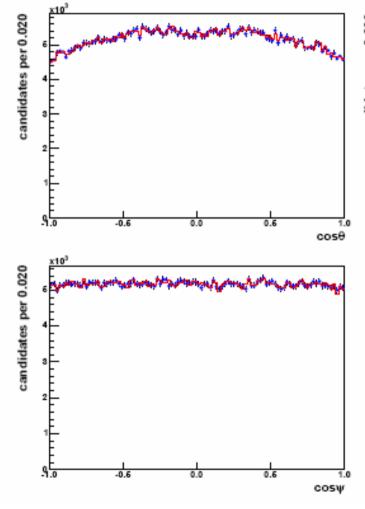
Short-lived

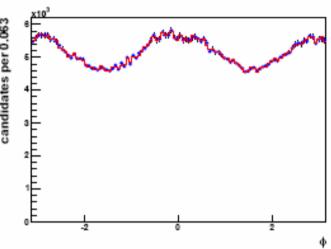
Majority of the background

- **Prompt J/\psi** paired with tracks (ct = 0)
- Combinations with mis-measured tracks (ct > 0, ct < 0)
- Background Angular Distribution (allow for S, P, D components)
- Correction for detector efficiency and acceptance

Detector Acceptance and Efficiency







- 40 M Full MC decays generated flat in angular variables
- Shapes show effect of cuts and detector sculpting
- This sculpting is corrected for by including an additional normalization term in the likelihood function
- Realistic MC tests and Pull tests ensure the correctness of the treatment.

B_d Fit: Mass and Lifetime Projection



Lumi = 258±15 pb⁻¹

ct(Sig) |ct(Bkg_{all})

ct(Bkg s)

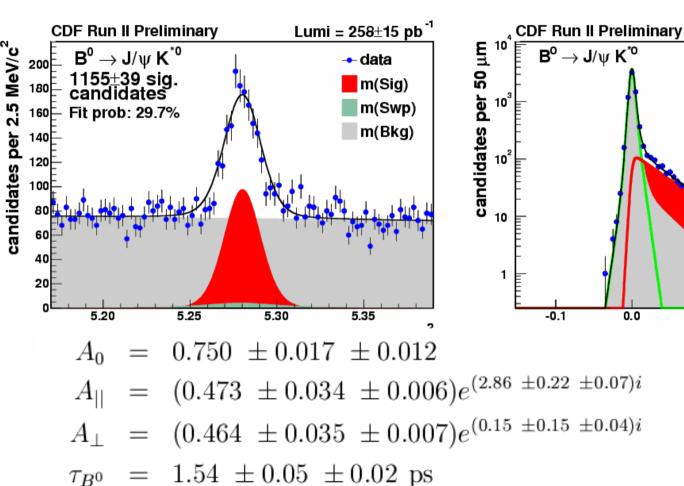
Fit prob: 60.7%

o.3 ct, cm

- data

0.2

0.1

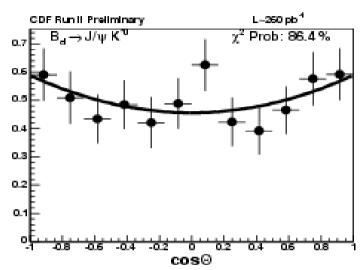


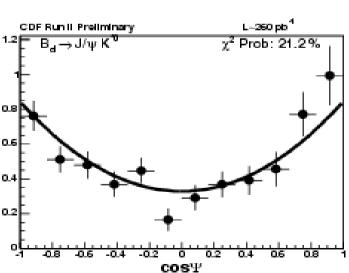
vs. PDG = 1.537 ± 0.015 ps

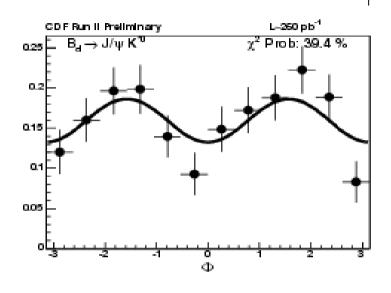
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Angular Projection (Bd)









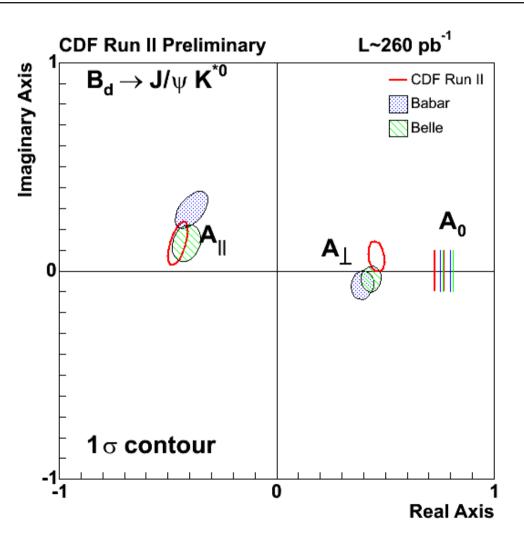
> Projections:

- Sideband subtracted
- Detector sculpting Corrected

Single variable projection

B_d Amplitudes vs. Babar/Belle





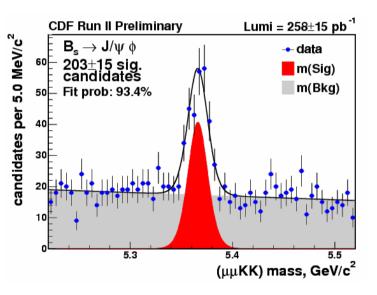
Babar PRL 87, 241801 (2001) Belle P.L B538, 11 (2002)

Bs Unconstrained Fit

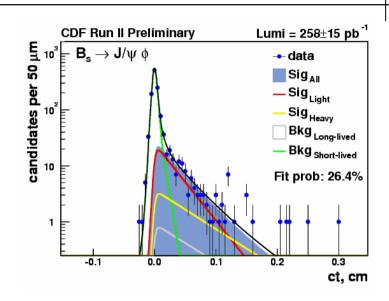


Fit data as described

Lifetime & Mass Projection



$$SM: \Delta\Gamma/\Gamma = 0.12 \pm 0.06$$



$$A_0 = 0.784 \pm 0.039 \pm 0.007$$

 $A_{||} = (0.510 \pm 0.082 \pm 0.013)e^{(1.94 \pm 0.36 \pm 0.03)i}$
 $|A_{\perp}| = 0.354 \pm 0.098 \pm 0.003$
 $\tau_L = 1.05^{+0.16}_{-0.13} \pm 0.02 \text{ ps}$
 $\tau_H = 2.07^{+0.58}_{-0.46} \pm 0.03 \text{ ps}$
 $\Delta\Gamma/\Gamma = 0.65^{+0.25}_{-0.33} \pm 0.01$
 $\Delta\Gamma = 0.47^{+0.19}_{-0.24} \pm 0.01 \text{ ps}^{-1}$

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Bs Constrained Fit



Theory: $\Gamma_s / \Gamma_d = 1.00 \pm 0.01$

DPF, 2004

Experiment : $c \tau_d = 460.8 \pm 4.5 \mu m$

$$\tau_s = \frac{2\tau_H \tau_L}{\tau_H + \tau_I} \xrightarrow{constrained} \tau_d$$

Constrain: $c\tau_s$ to $460.8 \pm 6.4 \mu m \ (\sqrt{4.5^2 + 4.6^2})$

Gaussian constraint in the likelihood fit

$$A_0 = 0.783 \pm 0.038 \pm 0.007$$

$$A_{||} = (0.539 \pm 0.070 \pm 0.013)e^{(1.91 \pm 0.36 \pm 0.03)i}$$

$$|A_{\perp}| = 0.308 \pm 0.087 \pm 0.003$$

$$\tau_L = 1.13^{+0.13}_{-0.09} \pm 0.02 \text{ ps}$$

$$\tau_H = 2.38^{+0.56}_{-0.43} \pm 0.03 \text{ ps}$$

$$\Delta\Gamma/\Gamma = 0.71^{+0.24}_{-0.28} \pm 0.01$$

$$\Delta\Gamma = 0.46^{+0.17}_{-0.18} \pm 0.01 \text{ ps}^{-1}$$

CDF Run II Preliminary

Lumi =
$$258\pm15$$
 pt

data

Sig AII

Sig Heavy

Bkg Long-lived

Bkg Short-lived

Fit prob: 37.8°

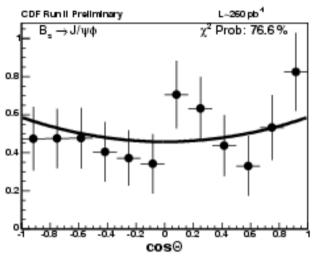
$$SM: \Delta\Gamma/\Gamma = 0.12 \pm 0.06$$

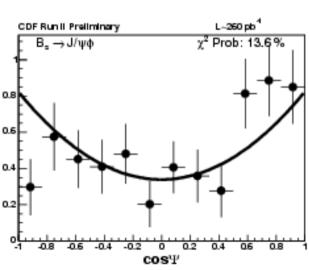
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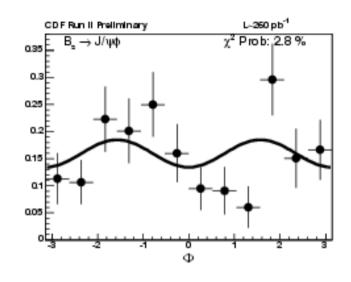
13

Angular Projection (Bs)









Projections:

- Sideband subtracted
- Acceptance Corrected

Single variable projection

Systematics

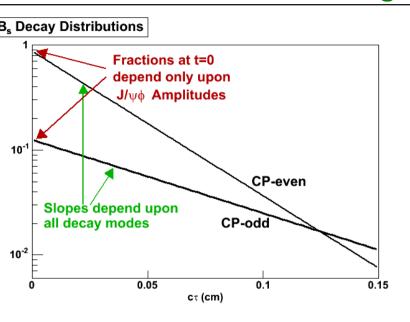


B_d	$c\tau$, μ m	$ A_0 $	$ m{A}_{ } $	$ A_{\perp} $	$arg(A_{\parallel})$	$arg(A_\perp)$
Bkg. ang. model	3.9	0.009	0.006	0.006	0.01	0.01
Eff. and acc.	_	_	_	_	_	_
$\mathbf{K} \leftrightarrow \pi$ swap	_	0.002	0.002	0.002	0.01	_
Non-resonant decays	_	0.007	0.001	0.004	0.07	0.04
Bkg. lft. model	1.7	_	_	_	_	_
SVX alignment	1.0	_	_	_	_	_
Lft. bias	1.3	_	_	_	_	_
B_s cross-feed	_	_	_	_	_	_
Total	4.6	0.012	0.006	0.007	0.07	0.04

B_s	$c au_L, \mu\mathrm{m}$	$\Delta\Gamma/\Gamma$	$ A_0 $	$ m{A}_{\parallel} $	$ m{A}_{\perp} $	$rg(A_\parallel)$
Bkg. ang. modeD	3.7	0.007	0.007	0.013	0.003	0.03
Eff. and acc.	_	_	_	_	_	_
Unequal $\#~B_s,ar{B}_s$	_	_	_	_	_	_
Bkg. Ift. model	1.7	_	_	_	_	_
SVX alignment	1.0	_	_	_	_	_
Lft. bias	1.3	_	_	_	_	_
$\bigcirc B_d$ cross-feed	5.0	0.008	_	0.003	0.001	_
Total	6.7	0.011	0.007	0.013	0.003	0.03

Cross Check: B_s CP odd fraction





Bs CP Odd

Cut (µm)	Fitted (%)	Predicted (%)
>0	20.1 +/- 9.0	20.1
>150	24.2 +/- 10.3	24.1
>300	29.6 +/- 12.7	28.6
>450	38.7 +/- 11.6	33.6

Bd P Odd

Cut (μm)	Fitted (%)
>0	21.6 +/- 4.4
>150	23.0 +/- 3.6
>300	23.0 +/- 4.0
>450	23.6 +/- 4.9

- Fit to amplitudes ONLY, using different minimum lifetime cuts.
 - The CP odd fraction increase in the B_s fit suggests significant lifetime difference in the two components
 - > The predictions of the fraction using our measured lifetime difference are consistent with the angular fitting results
- The CP odd fraction of B_d stays constant with different ct cuts. Consistent with our expectation.

Betting Odds



How likely are we to observe this value of $\Delta\Gamma_{\rm s}/\Gamma_{\rm s}$ if the true value were zero(or 0.12)?

- > 10000 Toy MC fits to estimate the probability of a fluctuation (with $\Delta\Gamma_{\rm s}/\Gamma_{\rm s}$ > our measurement)
 - ► P(measured | true = 0)
 - Unconstrained Fit: 0.65 1/315
 - Constrained Fit: 0.71 1/718
 - ► P(measured | true = 0.12)
 - Unconstrained Fit: 0.65 1/84
 - Constrained Fit: 0.71 1/204

Conclusions



- Time dependent angular analysis powerful tool
- ➤ Competitive $B_d \rightarrow J/\psi K^{*0}$ amplitudes measurements (agree well with BaBar/Belle)
- **→** B_d lifetime agrees well with PDG
- > ~200 $B_s \to J/\psi \Phi$ show evidence of lifetime difference in B_s system.
 - > For constrained fit, we measured:

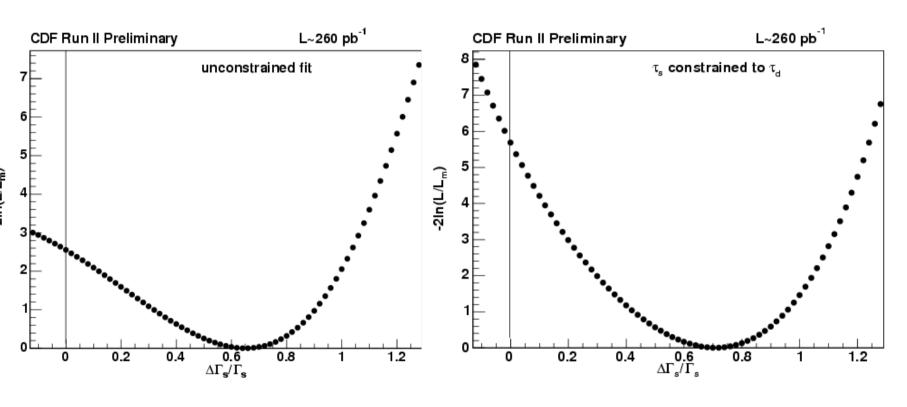
$$\Delta \Gamma_s = 0.46 \pm 0.18 \pm .01 \text{ ps}^{-1}$$
 $\frac{\Delta \Gamma_s}{\Gamma_s} = 0.71^{+0.24}_{-0.28} \pm 0.01$

- \rightarrow $\Delta\Gamma_s$ =0 ruled out at ~1/700 odds (with Γ_s = Γ_d constraints)
- > First measurement of B_s lifetime difference.
- More data coming underway!!! On the edge of Challenging Standard Model. $(\Delta\Gamma_s/\Gamma_s(SM) = 0.12 +/- 0.06)$

Backup Slides

Likelihood Scan





> Scan in $\Delta\Gamma_s/\Gamma_s$, refit at each point letting other parameters float

Main Fitting Results



	B_d	B_s Unconstrained Fit	B_s Constrained Fit	unit
M_B	5280.2 ± 0.8	5366.1 ± 0.8	5366.0 ± 0.8	MeV/c ²
A_0	0.750 ± 0.017	0.784 ± 0.039	0.783 ± 0.038	
$A_{ }$	0.473 ± 0.034	0.510 ± 0.082	0.539 ± 0.070	
A_{\perp}	0.464 ± 0.035	0.354 ± 0.098	0.308 ± 0.087	
$\delta_{ }$	2.86 ± 0.22	1.94 ± 0.36	1.91 ± 0.32	
δ_{\perp}	$\textbf{0.15} \pm \textbf{0.15}$			
$c\tau_B$	462 ± 15			μ m
$c au_L$		316 ⁺⁴⁸ ₋₄₀	340 ⁺⁴⁰ ₋₂₈	μ m
$c\tau_H$		622 ⁺¹⁷⁵ ₋₁₃₈	713 $^{+167}_{-129}$	μ m
$c\tau_s$		419 ⁺⁴⁵ ₋₃₈	460 ± 6.2	μ m
$\Delta\Gamma_s/\Gamma_s$		65 ⁺²⁵ ₋₃₃	71 +24	%
$\Delta\Gamma_s$		$0.47 \begin{array}{l} +0.19 \\ -0.24 \end{array}$	$0.46 \begin{array}{l} +0.17 \\ -0.18 \end{array}$	${\sf ps}^{-1}$
N_{sig}	1155 ± 39	203 ± 15	201 ± 15	

Other Fitting Parameters



Parameter	B_d result	B_s result	unit
f_s	0.151 ± 0.005	0.164 ± 0.012	
\overline{A}	-1.06 ± 0.89	-2.2 ± 1.2	$(\text{GeV/c}^2)^{-1}$
S_m	1.65 ± 0.06	1.81 ± 0.12	
$ B_0 ^2$	0.292 ± 0.009	0.318 ± 0.023	
$ B_{ } ^2$	0.358 ± 0.017	0.385 ± 0.041	
$arg(\ddot{B_{ }})$	1.60 ± 0.06	1.63 ± 0.13	
\ddot{f}_{-}	0.042 ± 0.014		
f_{+}	0.145 ± 0.019	0.124 ± 0.031	
f_{++}	0.044 ± 0.006	0.011 ± 0.007	
λ_{-}	47 ± 7		μ m
λ_{+}	45 ± 6	66 ± 17	μ m
λ_{++}	348 ± 40	634 ± 280	μ m
S_{ct}	1.27 ± 0.02	1.33 ± 0.04	
		·	

Implication for ∆m_s



$$\frac{\Delta\Gamma_s}{\Delta m_s}$$
 = $(3.7^{+0.8}_{-1.5})\times10^{-3}$ (B Physics at the Tevatron Report value) (Beneke, et al hep-ph/9808385 NLO analysis)

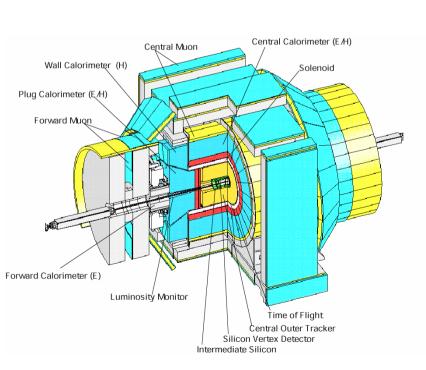
$$\Delta m_s = 125^{+69}_{-55} \, ps^{-1}$$

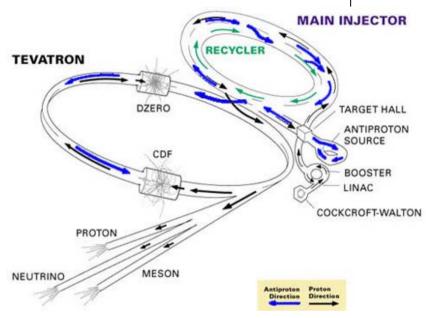
Using our constrained fit results

Current limit $\Delta m_s > 14.9 \text{ ps}^{-1}$ (95% C.L.)

Tevatron and CDF Run II

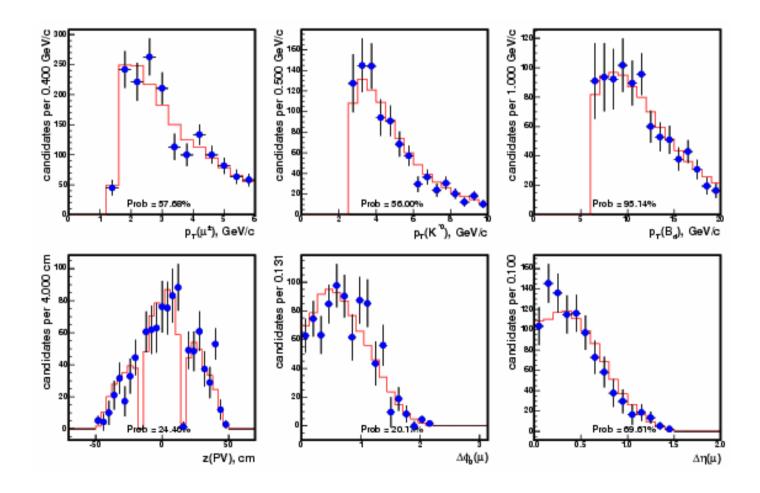






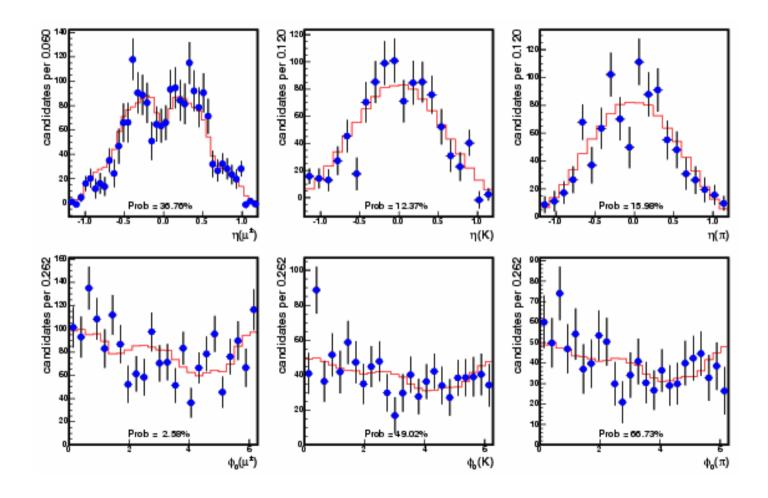
MC vs Data (B_d) (I)





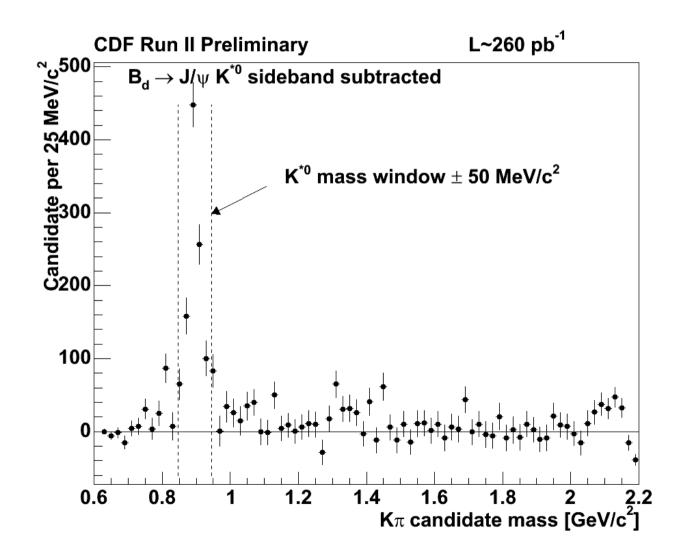
MC vs Data (B_d) (II)





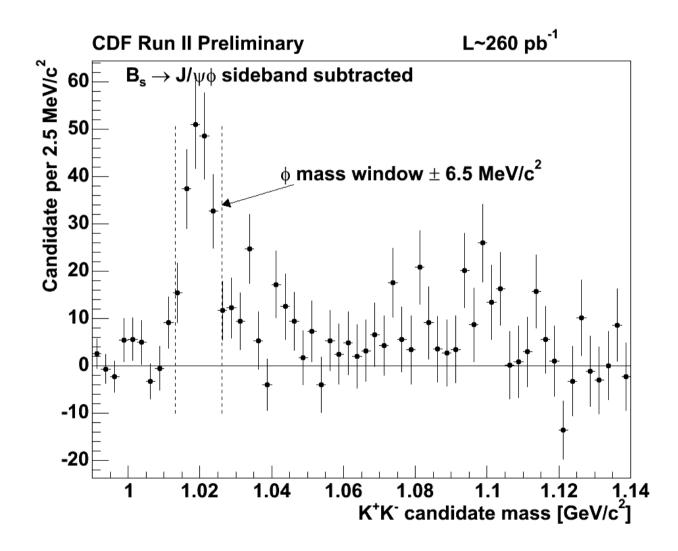
B_d Sideband Subtracted $K\pi$





B_s Sideband Subtracted KK





Detector Acceptance Correction



$$\Omega(\lbrace A_i \rbrace; \vec{\rho}) = \sum_{i} A_i f_i(\vec{\rho})
\Omega_{obs}(\lbrace A_i \rbrace; \vec{\rho}) = \Omega(\lbrace A_i \rbrace; \vec{\rho}) \varepsilon(\vec{\rho}) / \langle \varepsilon \rangle
= \sum_{i} A_i f_i(\vec{\rho}) \varepsilon(\vec{\rho}) / \langle \varepsilon \rangle$$

Log Likelihood function is

$$\begin{split} &\log L = \log \left[\frac{1}{\left\langle \varepsilon \right\rangle} \Omega(\{A_i\}; \vec{\rho}) \varepsilon(\vec{\rho}) \right] \\ &= \log \left(\Omega(\{A_i\}; \vec{\rho}) \right) + \log \left(\varepsilon(\vec{\rho}) \right) - \log \left(\left\langle \varepsilon \right\rangle \right) \end{split}$$

Does not depend on A_i , can be dropped in the minimization procedure.

$$\begin{split} \left\langle \varepsilon \right\rangle &= \int\limits_{\rho} d\vec{\rho} \left\{ \sum_{i} A_{i} f_{i}(\vec{\rho}) \varepsilon(\vec{\rho}) \right\} \\ &= \sum_{i} A_{i} \left\{ \int\limits_{\rho} d\vec{\rho} f_{i}(\vec{\rho}) \varepsilon(\vec{\rho}) \right\} \\ &= \sum_{i} A_{i} \xi_{i}, \end{split}$$

$$= \sum_{i} A_{i} \xi_{i}, \end{split}$$

$$\xi_{i} = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} f_{i}(\vec{\rho})$$

Calculate ξ_i from Monte Carlo.

Include them into the likelihood function.

Detector Acceptance Correction